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EXPERIMENTAL STUDY OF HEAT TRANSFER IN LAYERED COMPOSITES*

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Abstract—The heat-transfer characteristics of layered composites where the interface is parallel to the major heat flow direction is experimentally studied for low frequency periodic excitation. From the results, the problem of using steady equivalent thermal properties for unsteady heat conduction analysis is examined. It is concluded that the use of steady equivalent thermal properties for unsteady heat conduction gives good results only when the thermal conductivities of the two constituents do not differ widely.

The use of a model with two "diffusivities" for characterizing the heat-transfer problem for this specific geometry is also discussed in detail. One of the diffusivities accounts for the phase shift of the periodic excitation; the other accounts for attenuation.

NOMENCLATURE

C_1, C_1', amplitudes of temperature of the fundamental wave at depths x and $x+\Delta x$, respectively;
C_n, amplitudes of temperature for different frequencies, $n = 1, 2, 3 \ldots$;
C_p, specific heat per unit mass;
K_1, K_2, thermal conductivities of materials one and two, respectively;
K_{AV}, steady effective thermal conductivity;
n, number of harmonic wave, $n = 1, 2, 3 \ldots$;
t, time;
T, temperature;
V_{2T}, volumetric ratio of material two to the total volume;
x, distance along the interface of two materials from the excited temperature boundary;
y, distance normal to the interface of two materials;
Δx, distance between any two locations.

Greek symbols

α_A, α_P, diffusivities for attenuation and phase shift, respectively;
ρ_1, ρ_2, densities of materials one and two, respectively;
ϕ_1, ϕ_2, phase shifts of the fundamental wave at depths x and $x+\Delta x$, respectively;
ω, frequency.

INTRODUCTION

THE USE of "effective" properties to characterize steady state heat transfer in composites is well established. Excellent reviews on effective thermal conductivity by Nahas *et al.* [1], Gorring and Churchill [2] and Dulnev and Zarichnyak [3] show application of the concept to a wide variety of systems with widely differing constituent properties. The concept has a sound mathematical basis and can be used with confidence.

Unfortunately, for unsteady heat transfer such confidence is not possible. In fact, it is known that for some systems where the concept of effective properties is valid in steady heat transfer, it is not valid for unsteady heat transfer (Ben-Amoz [4], Horvay *et al.* [5], Manaker [6] and Kaczenski [7]). Thus when dealing with unsteady heat transfer involving composite materials an analyst is faced with the predicament of knowing that the use of effective properties (specifically an effective diffusivity) is valid in some cases and not in others. He may suspect that the procedure is valid when the constituent properties are of similar magnitude or when the excitation is mainly composed of low frequencies (Horvay *et al.* [5], Kaczenski [7] and Horvay [8]), but he will question how high the frequencies may be or how different the properties may be before the errors become significant. It is the purpose of this paper to provide some guidance in the matter. A comprehensive answer to the above questions is not yet possible since the problem of effective properties in unsteady heat transfer is very complicated and has received little attention.

The attack taken here is to experimentally study thermal wave propagation in a system where the conditions for using effective properties are generally

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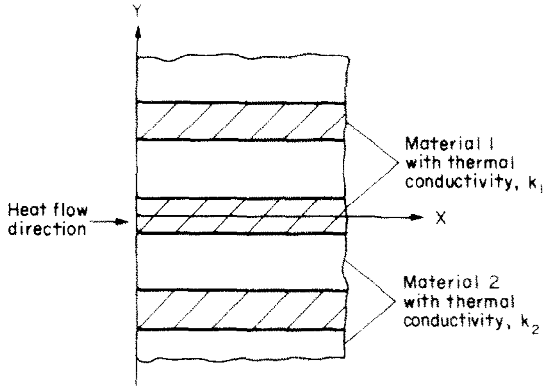

FIG. 1. Sketch of parallel alternate layer model.

unsatisfactory. It is a "worst" case. This study, in effect, bounds the conditions for which the use of effective properties is valid. The system considered is a laminated composite with the laminae running parallel to the principal heat flow direction as shown in Fig. 1. Another reason for choosing this system is that it is the most widely studied theoretically with respect to thermal wave propagation (Horvay *et al.* [5], Manaker [6], Kaczenski [7], Kaczenski and Horvay [8], Manaker and Horvay [9, 10] and Horvay and Manaker [11]). To the authors knowledge this is the first experimental study of the nature of unsteady heat transfer in this or other composite systems.*

The overall goal of the work is to characterize wave propagation in the laminated composite on the basis of an equal-weight average temperature at any plane perpendicular to the principal heat flow direction. An averaging technique which weights local temperatures according to laminae thickness or properties, while possibly more accurate, would be much more complicated than the model proposed here. As the wave moves into the composite it experiences both attenuation and phase shift which suggests that the composite can be better characterized by the use of two "diffusivities" rather than by a single diffusivity. One "diffusivity" would account for attenuation (α_A in this paper), and the other would account for phase shift (α_P in this paper). For homogeneous systems α_A and α_P are equal and can be obtained from either steady or unsteady experimental data. On the other hand, for composites, α_A is not equal to α_P though for low frequency excitation in composites with similar constituent thermal properties they are approximately equal. The question raised here and addressed by this research is how different α_A and α_P are from the thermal diffusivity calculated as the ratio of a steady effective conductivity, K_{AV}, and an average value of the heat capacity $(\rho C_P)_{AV}$. For a laminated composite as shown in Fig. 1 the effective conductivity is given by

$$K_{AV} = K_2 V_{2T} + K_1(1 - V_{2T}) \tag{1}$$

and the average heat capacity is given by

$$(\rho C_P)_{AV} = (\rho C_P)_2 V_{2T} + (\rho C_P)_1(1 - V_{2T}) \tag{2}$$

where K and (ρC_P) represent the thermal conductivity and heat capacity respectively. Subscripts 1 and 2 refer to the individual constituents. V_{2T} denotes the ratio of the volume of material 2 to the total volume. Thus

$$\alpha_{AV} = \frac{K_{AV}}{(\rho C_P)_{AV}}. \tag{3}$$

Significant deviation of α_A or α_P from α_{AV} will be an indication that a single diffusivity is not adequate in characterizing unsteady heat transfer in the specific composite under study.

THE EXPERIMENT

A test apparatus was constructed to create a boundary condition on the sample free surface which approximated a square wave. This was done by alternately impinging water from reservoirs at two different temperatures on the free surface of the sample for equal times. No attempt was made to obtain a sinusoidal wave or to perfect the square wave. A sinusoidal wave requires an elaborate control system and a perfect square wave is impossible. Actually neither is necessary since harmonic analysis of the actual resulting wave gives all the information needed and is a much easier procedure.

The samples were constructed of individual laminae of various thicknesses. Combinations of four different materials taken two at a time were used. The materials included three metals of different but relatively high thermal conductivity (aluminum alloy, brass 360 alloy, and low carbon steel) and a polymer of relatively low thermal conductivity (PMMA†). Values of the important properties are given in Table 1. These were determined experimentally rather than relying on published data which varies widely. Extreme care was taken in sample construction and instrumentation to reduce interface resistances, edge effects, thermocouple disturbances, etc. Thermocouples were embedded in the samples at three different depths from the free surface. The thermocouples at any one depth were connected in parallel to give an accurate output across the section. The first step in testing any sample after mounting in the test apparatus was to subject the sample to a sufficient number of cycles to obtain a steady periodic condition. Three periods of 90, 120 and 180 s were used. Average thermocouple responses at each of the three depths were then continuously recorded for several cycles after the steady periodic condition is reached.‡

Readers interested in details of any of the above procedures are referred to Truong [12].

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* Many researchers use unsteady methods to obtain effective thermal conductivities of composites with the tacit assumption that the procedure is valid. On the basis of the above discussion and the work herein, some of this work must be in error.

† Poly (methyl methacrylate).

‡ Individual temperature measurements were also made in several cases to determine the actual temperature profile to assure that this averaging procedure was valid.

Table 1. Properties of homogeneous materials used in this study

	Property				
Materials	Density [kg/m^3 × 10^{-3}]	Thermal diffusivity [m^2/s × 10^5]	Thermal conductivity [J/s·m·K × 10^{-2}]	Specific heat [J/kg·K × 10^{-2}]	Temperature range [K]
Aluminum 2024 alloy	2.79±0.02	4.11±0.21	1.094±0.066	9.55±0.017	300 ~ 311
Brass 360 alloy	8.47±0.03	2.53±0.05	0.818±0.024	3.82±0.021	300 ~ 311
Low carbon steel	7.84±0.03	1.39±0.05	0.488±0.019	4.56±0.004	300 ~ 311
PMMA	1.17±0.01	0.0116±0.0005	0.00182±0.00004	13.40±0.008	300 ~ 311

DATA ANALYSIS AND RESULTS

A computer program was used to convert the raw thermocouple output data to temperature data using previously developed calibrations. The program also performed a harmonic analysis to extract the fundamental wave. Only the fundamental wave is necessary since the higher harmonics attenuate very rapidly. It was confirmed that the error involved in neglecting the higher frequency terms is less than 2%. This fundamental wave was then analyzed as described next.

Because the objective is to establish effective properties of a composite corresponding to an "equivalent" homogeneous material, the starting point of the analysis is the equation for the temperature response at any depth from the free boundary of a semi-infinite homogeneous material subjected to a periodic boundary temperature. This response is [13]

$$T(x,t) = \sum_{n=1}^{\infty} C_n \exp\left[-\left(\frac{n\omega}{2\alpha}\right)^{1/2} x\right] \times \cos\left[n\omega t - \left(\frac{n\omega}{2\alpha}\right)^{1/2} x\right] \quad (4)$$

where C_n is the amplitude of temperature for any harmonic ($n = 1$ corresponds to the fundamental wave), ω is the frequency (period $= 2\pi/\omega$), α is the thermal diffusivity, x is the depth from the free boundary and t is time. Since the higher harmonics attenuate very rapidly and have negligible effect after only a short distance from the surface, these can be neglected with little error. Thus we assume that the "equivalent" homogeneous material has a response of the form

$$T(x,t) \simeq C_1 \exp\left[-\left(\frac{\omega}{2\alpha}\right)^{1/2} x\right] \times \cos\left[\omega t - \left(\frac{\omega}{2\alpha}\right)^{1/2} x\right]. \quad (5)$$

Using this model it is possible to calculate the equivalent thermal diffusivity, α, from the fundamental wave at two different depths using either the attenuation portion of the expression $\{C_1 \exp[-(\omega/2\alpha)^{1/2}x]\}$ or the phase shift portion $\{\cos[\omega t-(\omega/2\alpha)^{1/2}x]\}$. Using attenuation gives

$$\alpha_A = \frac{\omega \Delta x^2}{2\left[\ln\left(\frac{C_1'}{C_1}\right)\right]^2} \quad (6)$$

where C_1 and C_1' are the amplitudes of the fundamental wave at two different depths separated by a distance Δx. Using phase shift gives

$$\alpha_P = \frac{\omega \Delta x^2}{2(\phi_1' - \phi_1)^2} \quad (7)$$

where ϕ_1 and ϕ_1' are the phase shifts at two different depths. If, indeed, calculations show α_A and α_P to be approximately equal then a homogeneous equivalent does exist. If they are not equal, then the "equivalent" is a physically non-existent invention with two diffusivities, one to govern attenuation and one to govern phase shift.

To further help answer the question of when α_{AV} is a good approximation of α_A and α_P, the latter were normalized using α_{AV}. The representative results for steel/aluminum and aluminum/PMMA laminated composites are presented in Figs. 2 and 3 respectively where least square parabolas have been fitted to the

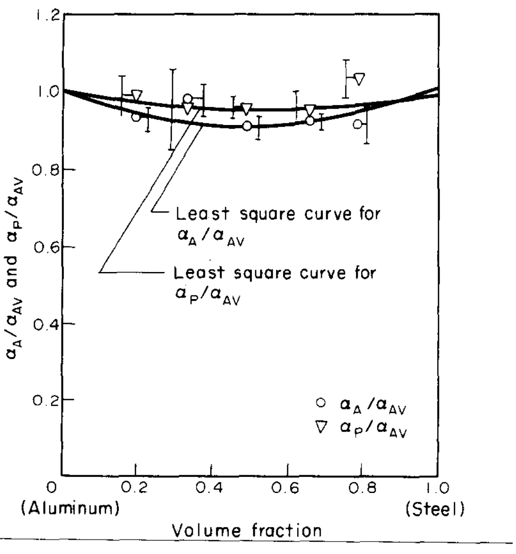

FIG. 2. Variation of α_A/α_{AV} and α_P/α_{AV} with volume fraction for low carbon steel/aluminum 2024 alloy.

data. It should be noted that all these least square parabolas very nearly go through the points $\alpha_A/\alpha_{AV} = \alpha_P/\alpha_{AV} = 1$ for volume fractions of 0 and 1 as they should since these volume fractions correspond to the homogeneous limits.

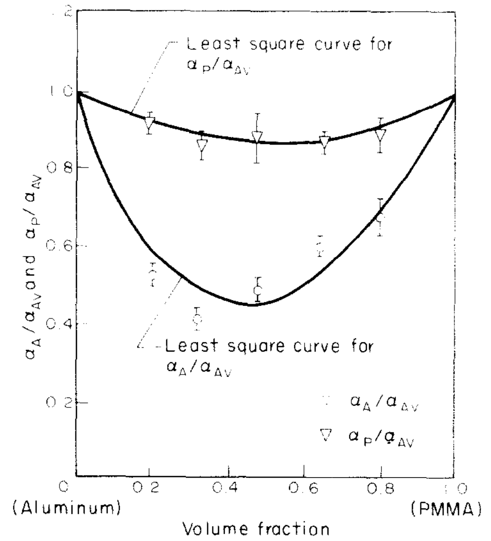

FIG. 3. Variation of α_A/α_{AV} and α_P/α_{AV} with volume fraction for aluminum 2024 alloy/PMMA.

The data points in Figs. 2 and 3 are actually the average of six data points. Since α_A and α_P showed no particular trend with frequency of the imposed surface temperature boundary condition, the data for the three frequencies were averaged. Similarly, α_A and α_P were calculated using two of the three possible combinations of thermocouple locations and no trend was noted, so these results were also averaged. Consequently each data point in Figs. 2 and 3 represent the average of the six possible combinations of three frequencies and two pairs of thermocouple locations.* The "error" bars in the figures indicate the spread of these data points measured by plus or minus one standard deviation.

DISCUSSION AND CONCLUSIONS

The independence of α_A and α_P with frequency is particularly noteworthy since this result confirms theoretical predictions (Horvay *et al.* [5], Kaczenski [7] and Kaczenski and Horvay [8]). These researchers noted that at low frequencies and not too small distances into the laminate, the static equivalent thermal constant α_{AV} is applicable also in non-static problem. Additionally, this research shows that even when α_{AV} is not applicable it is still possible in some cases to predict average temperatures by use of a model with two diffusivities, α_A and α_P which are also frequency independent. Likely this holds only for not too small distances into the laminate.

It is concluded from this study that metallic/metallic laminated composites subjected to low frequency excitation can be modeled quite accurately using α_{AV} for both phase shift and attenuation prediction. Even for the worst cases, the least square curves for the ratios of α_A/α_{AV} and α_P/α_{AV} fall well within 10% error though some individual data points are slightly out of this range. An analyst can then be quite comfortable when using α_{AV} for metallic composites whose constituent properties are similar to those used in this study. This would be true for most composites except in some laminates where the principal heat flow direction is perpendicular to the laminae. However, when the properties of the constituents are quite different as is the case when the metallic/PMMA laminated composites α_{AV} is a good predictor only of phase shift (since α_P/α_{AV} is close to one for all volume fractions). Attenuation prediction using α_{AV} would introduce large error since α_A/α_{AV} is close to one only for volume fractions close to zero or one (the homogeneous limits).

Composites with properties falling between the two extremes studied in this work would display curves of α_A/α_{AV} and α_P/α_{AV} vs volume fraction between those presented. An analyst could make predictions based on some interpolated values of α_A and α_P, but this procedure would be very approximate at best. We feel an analyst would be better off using α_{AV} and introducing a safety factor based on the information presented here.

There are some cases which should be the subject of further study to really cover the range of possible composites. Although the metallic/metallic composites had thermal conductivity ratios of approximately one the results are not obviously applicable to composites of very low thermal conductivity even if the thermal conductivity ratios are close to one. Also, the thermal conductivity range of 10–100 needs further study. The effect of frequency should also be investigated to determine the ranges of applicability of the single and double diffusivity models.† Since the details of this work as reported in Truong [12] have confirmed the analytical procedures we suggest that further work on modeling be analytical especially since high frequency experimental data is extremely difficult if not impossible to generate.

The double diffusivity model can apparently extend the range of applicability of the single diffusivity model. However, a method for predicting these two diffusivities is by no means clear. Correlation procedures such as presented in Truong [12] are unsatisfactory because of being too specific. The ultimate goal of any continuation of this work should be such a prediction method based on a solid mathematical

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*Readers interested in the six separate data points are referred to Truong [12] where this data and data for other laminated composites are preserved. This reference also discusses a correlation procedure for the results in Figs. 2 and 3 as well as the other laminated composites but the correlations are not presented here because they are too specific.

†Horvay and Manaker [11] present a theoretical analysis of the high frequency problem for the laminated composite of this study.

foundation. We have shown here that such a model is possible but because of the complexities involved, the prediction procedure eludes us.

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ETUDE EXPERIMENTALE DU TRANSFERT THERMIQUE DANS DES COMPOSITES STRATIFIES

Résumé—On étudie expérimentalement, pour une excitation périodique de faible fréquence les caractéristiques du transfert thermique de composites en couches dont l'interface est parallèle à la direction principale du flux de chaleur. A partir des résultats, on examine le problème de l'utilisation des propriétés équivalentes en régime permanent au cas de la conduction thermique variable. On conclut que cette utilisation donne de bons résultats seulement quand les conductivités thermiques des deux constituants ne diffèrent pas beaucoup. On discute aussi en détail un modèle à deux "diffusivités" pour caractériser le problème dans cette géométrie spécifique. Une de ces diffusivités tient compte de la phase de l'excitation périodique et l'autre tient compte de l'atténuation.

EINE EXPERIMENTELLE STUDIE ZUR WÄRMEÜBERTRAGUNG IN GESCHICHTETEN VERBUNDWERKSTOFFEN

Zusammenfassung—Die Wärmeübertragungseigenschaften von geschichteten Verbundwerkstoffen, bei denen die Trennfläche parallel zur Hauptrichtung des Wärmestroms verläuft, wurden experimentell für periodische Beaufschlagung bei niedrigen Frequenzen untersucht. Anhand der Resultate wurde die Möglichkeit geprüft, äquivalente stationäre thermische Stoffwerte für die Berechnung der instationären Wärmeleitung zu verwenden. Dabei ergab sich, daß die Verwendung äquivalenter stationärer thermischer Stoffwerte bei der instationären Wärmeleitung nur dann zu guten Resultaten führt, wenn die Wärmeleitfähigkeiten der beiden Bestandteile sich nicht zu sehr unterscheiden. Ausführlich wird die Brauchbarkeit eines Modells mit zwei Leitfähigkeiten zur Beschreibung der Wärmeübertragungseigenschaften dieser speziellen Geometrie diskutiert. Die eine Leitfähigkeit bestimmt dabei den Phasenverzug, die andere das Dämpfungsverhältnis des periodisch erregten Systems.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ТЕПЛООБМЕНА В СЛОИСТОЙ СТРУКТУРЕ

Аннотация — Теплообменные характеристики слоистых структур, в которых граница раздела параллельна основному направлению теплового потока, исследуются экспериментально при периодических низкочастотных возбуждениях. На основании полученных результатов рассматривается вопрос об использовании стационарных эквивалентных теплофизических свойств для анализа нестационарной теплопроводности. Показано, что это допустимо лишь в том случае, если теплопроводности двух компонент не сильно отличаются. Детально рассматривается использование модели с двумя коэффициентами температуропроводности для данной специфической геометрии. Один из коэффициентов температуропроводности учитывает фазовый сдвиг при периодической возбуждении, другой — затухание.